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Option implied ambiguity and its information content: Evidence from the subprime crisis

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Abstract This paper studies option investors' tendency to deviate from risk-neutrality around extreme financial events. We incorporate ambiguity into Black–Scholes theory and analyze the lead–lag association between option and stock markets during 2006–2008. Our findings from the Standard and Poor's 500 index options reveal that investors' option implied ambiguity moderates the lead–lag relationship between implied and realized volatility. We find that implied ambiguity contains predictive realized volatility information (beyond constant and stochastic implied volatilities), and that implied volatility is a less biased predictor of realized market variance when accounting for ambiguity in option pricing. We are also able to track changing investors' ambiguity perceptions (pessimism or optimism) prior to severe volatility events and document shifts in ambiguity aversion among put option holders in the period leading to the fall 2008 global market crash. Our results hold under multiple-priors and Choquet ambiguity specifications.

Keywords Choquet utility · Multiple-priors · Option implied ambiguity · Implied volatility · Realized volatility · Uncertainty

Abbreviations

BS	Black–Scholes
BSIV	Black–Scholes risk-neutral implied volatility
BSIV × IC	Interaction between BSIV and IC
CBOE	Chicago board options exchange
CDS	Credit default swaps

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CEU	Choquet expected utility
IC	Option implied ambiguity
ICBSIV	Ambiguity-adjusted implied volatility (BSIV × IC)
II	Investors intelligence
IV	Implied volatility
IV_c	Ambiguity-based implied volatility
MEU	Multiple-priors expected utility
NW	Newey–West
OTM	Out of the money
RV	Realized volatility
s × BSIV	Choquet-based implied volatility
SPX	S&P 500 index options
SV	Stochastic volatility
SV × IC	Interaction between SV and IC
VIX	CBOE implied volatility index

"In abnormal times…when the hypothesis of an indefinite continuance of the existing state of affairs is less plausible than usual…the market will be subject to waves of optimistic and pessimistic sentiment, which are unreasoning and yet in a sense legitimate where no solid basis exists for a reasonable calculation."

Keynes (1936)

1 Introduction

Current evidence documents a lead–lag association between the volatilities implied by prices of exchange-traded options and the volatilities realized subsequently in underlying asset prices (e.g., indexes and equities) (Whaley 2009; Clark and Baccar 2015). Options market prices are known to anticipate future underlying asset price behavior such that option implied volatility (IV) often acts as an efficient (though perhaps biased) predictor of future stock/index returns and realized volatility (Elkhodiry et al. 2011; Zagst and Kraus 2011). The CBOE's VIX index, IV and other options market information can be used to gauge market fear, serving as good examples of how to take stock of the informational efficiency of options markets and exploit interactions between derivative and primitive asset transactions. However, although providing relevant insights with respect to investors' expectations and risk aversion (Jackwerth 2000; Poteshman 2001; Kim and Leung 2014), these tools do not adequately explain investment behavior under severe uncertainty, ignorance or ambiguity conditions (Feldman 2007; Montesano 2008; Stiglitz 2011). Swings in investor opinions and market sentiment which normally occur around financial crashes or major economic shocks are not fully captured by standard options markets indicators. Risk monitoring measures such as VIX, Black–Scholes implied volatility (BSIV) and their variants fail to consider that individuals might often assign different decision weights to events with negative or positive realizations and that investors need not apply the same probabilistic decision rules to gains and losses when faced with uncertainty (Tversky and Kahneman 1992; Mandelbrot and Hudson 2008). As a consequence, estimations of options returns and implied volatilities are subject to "vagueness" and partial ignorance, causing miscalibration and "irrational" behavior (Knight 1921; Keynes 1936). Accounting for these subjective traits and the uncertainty aversion of individual investors, such as analysts and traders, can therefore provide more useful information about the effects of abnormal uncertainty on market sentiment, help detect signs of financial market instability, and potentially lead to better anticipation and monitoring of financial market swings.

In this paper, we investigate the uncertain behavior of US index put option holders during the (pre-)crisis and credit crunch period 2006–2008, highlighting the incremental information content of investors' option implied ambiguity (IC) in explaining the lead–lag relationship between option implied volatility and (realized) asset volatility (RV) under Knightian uncertainty conditions.[1] We contribute to the extant literature on uncertainty, derivatives and risk prediction by (1) showing evidence of ambiguity in the US index options market during 2006–2008, and (2) measuring the effect of ambiguity, implied directly from observed option prices, on realized index volatility. We find that, after controlling for implied volatility effects, ambiguity from derivatives markets predicts and is positively associated with stock market volatility in uncertain times. This effect has not been observed in prior financial economics and decision-making research.

We extend the Black–Scholes (BS) (1973) setting by incorporating ambiguity through Choquet Brownian motions, multiple-priors and the notion of c-ignorance (see e.g., Chateauneuf et al. 1996; Agliardi and Sereno 2011; Araujo et al. 2012) to the option pricing apparatus. We then use our extended model to empirically assess the informational efficiency of option implied volatility and ambiguity as predictors of stock index realized volatility. To this aim, we use a set of US SPX index put options (options written on the S&P 500 index) during 2006–2008, over a period of 588 trading days characterized by unusual uncertainty. Encompassing the fall 2008 global market crash, this time window provides a good 'laboratory' setting for gauging changing market sentiment and investors' heterogeneous beliefs in conditions of high uncertainty. Put options, as a form of insurance, are particularly suitable to capture investors' crash worries and risk concerns during the credit crunch and its aftermath.

We find that in times of high uncertainty, such as the 2006–2008 period, ambiguity-adjusted implied volatility (ICBSIV) backed-out from our extended option pricing model is a better predictor of stock market volatility than risk-neutral implied volatility (constant and stochastic). We also show that the ambiguity perceptions implied from index option prices contain incremental information beyond standard implied volatility, and document shifts in ambiguity aversion among US SPX index put option holders in the period leading up to the fall 2008 crash. Besides confirming a positive moderating role of investors' ambiguity aversion in explaining the lead–lag IV–RV relationship, we provide evidence on the information efficiency of options markets for a number of behavioral traits (e.g., pessimism and optimism) extracting investment sentiment and implied ambiguity directly from observed options prices. Option contracts not only contain information about the volatility/risk implied in options prices but also help characterize the ambiguity aversion or degree of investor confidence about future market prospects. This can help monitor swings in investor opinion and market sentiment not captured by standard risk models.

A number of studies have attempted to address related information efficiency issues using fuzzy principles of uncertainty and ambiguity (Cherubini 1997; Anderson et al. 2009;

[1] Here we view Choquet ambiguity as a type of Knightian uncertainty, considering ambiguity as a dimension of uncertainty beyond probabilistic risk that can be estimated under a partial ignorance framework using Choquet expected utility (CEU) and Choquet Brownian motions. The words (Knightian) uncertainty and ambiguity are used interchangeably (De Palma et al. 2008; Guidolin and Rinaldi 2013). Alternative frameworks for representing ambiguity include multiple-priors expected utility (MEU) (e.g., Nishimura and Ozaki 2007; Riedel 2009) and robust control theory (e.g., Liu et al. 2005; Marzban et al. 2015). Throughout the paper, we study multiple-priors ambiguity as a special case of Choquet uncertainty.

Jahan-Parvar and Liu 2014). Other authors examine problems pertaining to model uncertainty (Buraschi and Jiltsov 2006; Li 2007), heterogeneous beliefs (Liu et al. 2005; Han 2008) and market incompleteness (Mellios 2007; Beber et al. 2010) using analysts forecasts, expert survey data or other proxy information. Explicitly focused on derivatives transactions, Boyarchenko (2012) and Drechsler (2013) investigate how underlying realized volatility causes model uncertainty or ambiguity in CDS and option markets. In a similar vein, Polkovnichenko and Zhao (2013) show how probability weighting might explain option investors' behavior in the US.[2] None of these papers has investigated empirically the information content of ambiguity aversion in the lead–lag option and stock markets association. Addressing this gap, we introduce the concept of option implied ambiguity to the literature and demonstrate its predictive power regarding future volatility. We examine theoretically and assess empirically the information role of ambiguity aversion in index options prices. This is achieved by analyzing the lead–lag IV–RV relationship under both Choquet and multiple-priors ambiguity specifications.

We use Choquet expected utility (CEU) as a general framework to represent ambiguity as model parameter uncertainty generally affects the first and second moments of the distribution of index returns in times of ambiguity (e.g., Gollier 2008; Ford et al. 2014). We rely on previous evidence suggesting that several phenomena under ambiguity can be explained under the CEU lens (e.g., Kelsey et al. 2011; Nguyen et al. 2012). Moreover, the CEU framework is nested within cumulative prospect theory, implying that individuals attach different weighted probabilities (capacities) to various events depending on whether they result in losses or gains under uncertainty. The basic asymmetric nature of options fits well within this context. We examine the multiple-priors expected utility (MEU) specification for comparison as it is a special case of CEU plus it has been empirically validated in a number of empirical settings (Hey et al. 2010).

We extend the existing literature on the global financial crisis, incomplete information and option-based risk prediction by extracting ambiguity from observed put options prices given our uncertainty-based option model, and unveil the incremental information content of option implied ambiguity in explaining the realized dispersion of index returns in the US for the turbulent 2006–2008 period. We show that ambiguity aversion, in the form of option holders' subjective deviations from risk-neutrality, is positively associated with index volatility in uncertain times and that option implied ambiguity contains predictive realized volatility information beyond standard risk-neutral IV and other option implied information (e.g., the CBOE VVIX and SKEW indices).

Our findings underscore the need to consider broader models of uncertainty for tracking and monitoring market volatility during periods of economic and financial instability. The remainder of the paper is organized as follows. Section 2 presents the economic setup and modeling framework used to describe ambiguity and develops our ambiguity-based put option pricing model. Section 3 describes our data and empirical methods. Section 4 discusses our empirical findings and implications. Section 5 concludes.

2 Background and modeling framework

Prior research dealing with miscalibration and model misspecification has addressed the problem of option pricing and investment under ambiguity from several perspectives, such

[2] We go beyond descriptive observations by measuring and highlighting empirically the information content of investors' ambiguity aversion, via the IV–RV linkage, in derivatives markets.

as robust control (Trojani and Vanini 2004; Liu et al. 2005; Cogley et al. 2008; Jaimungal and Sigloch 2012), multiple-priors (Nishimura and Ozaki 2007; Riedel 2009; Vorbrink 2011; Faria and Correia-da-Silva 2012; Ben Ameur and Prigent 2013) and Choquet ambiguity (Chateauneuf et al. 1996; De Waegenaere et al. 2003; Javanmardi and Lawryshyn 2015). Not many studies, however, have been devoted to the key issue of risk prediction and how derivatives markets ambiguity affects stock market volatility assessment. In financial markets, ambiguity refers to situations where market participants are not sure about the likelihoods of the states of the world (e.g., resulting in ambiguous rates of return and volatility). Such ambiguous situations, when they occur, affect every asset class in the marketplace (Trojani and Vanini 2004; Faria and Correia-da-Silva 2012). This study investigates such a phenomenon, the 2006–2008 subprime crisis, through the informational lens of options markets. Our main hypothesis is that due to partial ignorance and model misspecification among investors, ambiguity aversion should be positively related to stock market variance in uncertain times, and that option implied ambiguity contains predictive information about realized volatility beyond that of implied volatility.

We examine the role of ambiguity in US derivatives markets during the subprime crisis and reassess the efficiency of options pricing information in volatility prediction. We revisit the fundamental relationship between option and stock markets from the perspective of Choquet and multiple-priors ambiguity (Gilboa and Schmeidler 1989, 1994; Schmeidler 1989), and provide robust evidence on the (incremental) predictive power of option implied ambiguity regarding future underlying volatility. We further show that the lead–lag IV–RV relationship goes beyond risk-neutrality and holds under different degrees of ambiguity, disproving the superiority of the standard risk-neutral IV as an efficient predictor of RV.

A few studies have recently dealt with the dynamics of option pricing under Choquet expected utility. These include Muzzioli and Torricelli (2004) on option pricing in illiquid markets, and Kast and Lapied (2010) and Roubaud et al. (2010) on investment timing and optimal stopping. The CEU framework fits well in a market environment where complex instruments are traded by "sophisticated" investors. We test, in comparison to MEU, the extent to which investors follow sophisticated decision-rules (such as CEU) in dealing with ambiguous prospects. This is an alternative to simpler rules such as the maxmin (multiple-priors) or maxmax criteria (Dana 2002). Following Chateauneuf et al. (2001, 2007), Kast and Lapied (2010), Kast et al. (2014) and Driouchi et al. (2015), we develop a general European option pricing model applied to the valuation of index put options and use it to extract option implied ambiguity and implied volatility estimates under uncertainty. We then assess the information content of index put option prices[3] over the 2006–2008 subprime crisis, a period that covers both the 2007–2008 credit crunch and the 2008 global markets crisis. Characterized by unusual periods of uncertainty with pessimistic and optimistic swings in global markets, this time window presents a real-life 'experimental' environment to gauge investors' changing ambiguity perceptions.

We derive the generalized price of a European put option under ambiguity using a modified pricing kernel (e.g., Franke et al. 1999; Driouchi et al. 2015). We consider a two-asset economy in which investors' estimations of expected asset returns and variances are affected by "vagueness" and partial ignorance induced by model parameter uncertainty (e.g., Gollier 2008; Buraschi and Jiltsov 2006). The underlying process driving option prices is subject to perturbations of the drift and volatility components, so that investors might assign different

3 Put options are examined in this research because they represent a form of insurance against losses for investors and are, therefore, suitable for our study of ambiguous behavior in uncertain times. Although our qualitative conclusions hold for call options holders, studying call option investments is out of the scope of this paper.

probabilistic weights to potential gains or losses. The price of the underlying index S, on which the option is written, is assumed to follow the set of Choquet Brownian motions (e.g., Kast and Lapied 2010; Agliardi and Sereno 2011; Kast et al. 2014) of the form:

$$\frac{dS}{S} = (\mu + m\sigma)dt + s\sigma dz \quad (\forall m \in]-1, 1], \forall s \in]0, 1]) \tag{1}$$

where m and s (both functions of a probability weighting function or ambiguity proxy c), are the mean and standard deviation of a general Wiener process W, with $dW = mdt + sdz$ (such that $\frac{dS}{S} = \mu dt + \sigma dW$), with z being a standard Wiener process. This category of Brownian motions has been validated by Kast and Lapied (2010) and Kast et al. (2014) in the context of decision theory under ambiguity. The authors show that it satisfies dynamic consistency and rectangularity principles, can be obtained as the limit of a random walk and nests multiple-priors ambiguity where $m \neq 0$ and $s = 1$. The multiple-priors form captures optimism or pessimism in the drift term of the lognormal diffusion (but not in the volatility term). When $m = 0$ and $s = 1$, Eq. (1) simplifies to a standard (risk-neutral) formulation with indifference towards uncertainty. In the Choquet setup, behavioral and ambiguity-related variables m and s are determined by a weighted probability function or capacity variable c, with $0 < c < 1$. Based on the weights and degree of confidence about probabilistic judgment, this conveys whether individuals are seeking uncertainty or not. Eq. (1) is thus a stochastic Brownian process with model parameter uncertainty in both its drift and volatility components indicative of investors' model misspecification (swings in perceptions) under ambiguity.[4]

This representation of uncertainty follows a symmetric random walk which converges to a general Wiener process W with mean $m = 2c - 1$ and variance $s^2 = 4c(1-c)$, as in Agliardi and Sereno (2011), Kast and Lapied (2010) and Kast et al. (2014). It is a generalization of the MEU case which we also study for comparison (see, e.g., Trojani and Vanini 2004; Faria and Correia-da-Silva 2012 for some of the asset pricing implications of multiple-priors ambiguity). There is a range of possible values (determined by c) for the drift and volatility terms of the Brownian motions resulting in multiple probabilities of option exercise, heterogeneous behavior and imperfect hedging. In this sense, $m > 0$ ($m < 0$) signifies investor optimism (pessimism) and $s \neq 1$ means volatility is measured subjectively, generally underestimated, because of behavioral factor s. Since m and s are both determined by the capacity variable c, the latter is usually viewed as a composite indicator of investor ambiguity and uncertainty attitudes. Comparable to k-ignorance, c has been used as a measure of uncertainty attitude or degree of confidence about probabilistic judgment in decision-making and behavioral economics (Eichberger and Kelsey 1999; Wakker 2001; Araujo et al. 2012). Acting as a conditional capacity in the Choquet integral, c summarizes decision-makers' ambiguity perceptions about index prices, with $0 < c < 0.5$ representing degrees of ambiguity aversion, while $0.5 < c < 1$ implying ambiguity-seeking behavior. $c = 0.5$ reduces to the traditional probabilistic framework (i.e., risk or ambiguity-neutral case). $c < 0.5$ ($c > 0.5$) implies that higher decision weights are assigned to choices with pessimistic (optimistic) outcomes. We find that, as an indicator for option investors' tendency to deviate from risk-neutrality, the c measure is significantly correlated with the US consumer confidence index (CCI) and the US policy uncertainty index (PUI) (both viewed as proxies for uncertainty in the economics literature) during the 2006–2008 period. We also find that c is significantly positively associated with (and predicts) the difference between RV and IV. This difference would not exist in the absence of uncertainty. Therefore, c is a reasonable proxy for ambiguity and uncertainty attitudes in our option pricing context.

[4] Equation (1) also allows for more than one asset price S in the economy under ambiguity and implies uncertainty in bid and ask spreads (De Waegenaere and Wakker 2001).

Let B be the price of a riskless bond with instantaneous rate of return r such that:[5]

$$\frac{dB}{B} = rdt \tag{2}$$

Let P be the price of a contingent-claim (e.g., a European put option on the stock index) which depends only on S and time t, $P(S,t)$. From Ito's lemma and Eq. (1), the dynamics of option price P under ambiguity can be written as:

$$\begin{aligned} dP(S,t) &= \frac{\partial P}{\partial t}dt + \frac{\partial P}{\partial S}\left[(\mu + m\sigma) Sdt + (s\sigma) SdZ\right] \\ &\quad + \frac{1}{2}\frac{d^2P}{dS^2}\left([(s\sigma)SdZ]^2 + [(\mu + m\sigma)Sdt]^2 + [(s\sigma)SdZ \times (\mu + m\sigma)Sdt]\right) \end{aligned} \tag{3}$$

This simplifies to:

$$dP(S,t) = \left[\frac{\partial P}{\partial t} + \frac{\partial P}{\partial S}(\mu + m\sigma) S + S^2 \frac{1}{2}\frac{d^2P}{dS^2}(s\sigma)^2\right] dt + \frac{\partial P}{\partial S}(s\sigma) SdZ \tag{3'}$$

Because of the distorted nature of Eq. (1), c perturbs both the drift and volatility components of Eq. (3′). Having specified the general price dynamics of securities S, B and P, we next identify the related market pricing kernel and apply martingale theory (Doob 1953) to arrive at the fundamental valuation equation for derivative price P under ambiguity.

The level of marginal utility in the economy ξ follows the Choquet ambiguity dynamics:[6]

$$\frac{d\xi}{\xi} = [mg(\xi,S) + f(\xi,S)]\,dt + sg(\xi,S)\,dz \quad (\forall m \in]-1,1[, \forall s \in]0,1]) \tag{4}$$

Then

$$\begin{aligned} d(\xi B) &= \xi(rBdt) + B\left[(mg(\xi,S) + f(\xi,S))\,\xi dt + sg(\xi,S)\,\xi dz\right] \\ &= \xi B\left[(r + mg(\xi,S) + f(\xi,S))\,dt + sg(\xi,S)\,\xi dz\right] \end{aligned} \tag{5}$$

The drift (dt) being zero implies:

$$r + mg(\xi,S) + f(\xi,S) = 0 \quad \text{or} \quad f(\xi,S) = -r - mg(\xi,S) \tag{6}$$

Following a similar process for S:

$$d(\xi S) = \xi dS + Sd\xi + d <\xi, S> \tag{7}$$

where in the last term in Eq. (7), $<,>$ stands for the inner product of functions ξ and S.

$$\begin{aligned} d(\xi S) &= \xi S\left[(\mu + m\sigma)\,dt + s\sigma dz\right] + S\xi\left\{[mg(\xi,S) - r - mg(\xi,S)]\,dt \right. \\ &\quad \left. + sg(\xi,S)\,dz\right\} + s^2\sigma\xi Sg(\xi,S)\,dt \\ &= \xi S\left[(\mu + m\sigma) - r + s^2\sigma g(\xi,S)\right] dt + S\xi\left[s\sigma + sg(\xi,S)\right] dz \end{aligned} \tag{8}$$

[5] We assume, for simplicity, that ambiguity does not yet have an impact on equilibrium interest rates. This corresponds to cases where shocks in S_t are not yet correlated ($\rho_2 = 0$) to those of the economic output rate or where the latter is simply deterministic, as highlighted by Faria and Correia-da-Silva (2012) in their general equilibrium framework under ambiguity.

[6] This results from $\frac{d\xi}{\xi} = f(\xi,S)\,dt + g(\xi,S)\,dW$ (see Harrison and Kreps 1979) and the characteristics of W in the Choquet ambiguity universe. The functions g and f help derive the ambiguity-adjusted formula for the pricing kernel.

This implies:

$$(\mu + m\sigma) - r + s^2\sigma g(\xi, S) = 0 \quad \text{or} \quad g(\xi, S) = \frac{[r - (\mu + m\sigma)]}{s^2\sigma} \tag{9}$$

Equation (9) is akin to an ambiguity-adjusted Sharpe ratio (see also Gollier and Schlee 2011; Rieger and Wang 2012). Ambiguity appears in both the numerator and denominator of this equation. Excess returns are corrected by an ambiguity-related factor $m\sigma$ while the risk component is adjusted by ambiguity scalar s. The excess returns correction is in line with the multiple-priors findings of Trojani and Vanini (2004) (i.e., their Proposition 3). The risk adjustment matches their MEU-unrelated robust control setting (i.e., their Eq. (17)). Our Choquet specification (Eq. 9) accounts for both corrections. It is noteworthy that $m\sigma$ from the MEU version of our Eq. (9) ($m \neq 0$ and $s = 1$) coincides with the market price of ambiguity (case where $\rho = 0$) derived by Faria and Correia-da-Silva (2012) in their multiple-priors-based general equilibrium model of asset prices.

We assume the pricing kernel follows the general structure of Harrison and Kreps (1979) dynamics but is affected by ambiguity in its fundamental component. Since they depend on ambiguity parameters m and s (with $-1 < m < 1$ and $0 < s \leq 1$), f and g are indeed not unique in this setting. This means that the pricing kernel captures swings in fundamentals but not pure sentiment (see Cochrane 2001; Shefrin 2005 and Han 2008 for discussions of how the marginal rates of substitution can be disconnected from the market kernel under uncertainty). Relaxing this market incompleteness or imperfect hedging assumption incrementally takes us back to the perfect replication or risk-neutral case of a Black and Scholes (1973) framework with constant volatility. Using the results from Eqs. (6) and (9):[7]

$$\begin{aligned}\frac{d\xi}{\xi} &= f(\xi, S)\,dt + g(\xi, S)\,dz\\ &= -r - m\left\{\frac{[r - (\mu + m\sigma)]}{s^2\sigma}\right\}dt + \left(\frac{[r - (\mu + m\sigma)]}{s^2\sigma}\right)dz\end{aligned} \tag{10}$$

Consider now the value of a put option P written on underlying stock index S. We derive the general case with dividend yield δ.[8]

$$\begin{aligned}d(\xi P) &= \xi dP + Pd\xi + d < \xi, P >\\ &= \xi\left\{\left[\frac{\partial P}{\partial t} + \frac{\partial P}{\partial S}(\mu - \delta + m\sigma)S + S^2\frac{1}{2}\frac{d^2P}{dS^2}(s\sigma)^2\right]dt + \frac{\partial P}{\partial S}(s\sigma)Sdz\right\}\\ &\quad + \xi P\left[-r - m\left\{\frac{[r - (\mu + m\sigma)]}{s^2\sigma}\right\}dt + \left(\frac{[r - (\mu + m\sigma)]}{s^2\sigma}\right)dz\right]\\ &\quad + \xi\left[\left(\frac{[r - (\mu + m\sigma)]}{s^2\sigma}\right)\frac{\partial P}{\partial S}(s\sigma)Sdt\right]\end{aligned} \tag{11}$$

Setting the drift (dt) term of the derivative to zero results in the ambiguity-adjusted fundamental equation for pricing contingent-claims (including the put option P):

7 This implies that $mg(\xi, S)\,dt + (s - 1)\,g(\xi, S)\,dZ = 0$, and that the market kernel is not equal to the marginal utility level. This results from market incompleteness that occurs during depressions or when the states of the world are not known (perfect hedging is no longer feasible under such conditions).

8 δ is introduced in the dt component of Eq. (1), replacing the drift term with $\mu - \delta$.

$$\xi\left[\frac{\partial P}{\partial t}+\frac{\partial P}{\partial S}\left(\mu-\delta+m\sigma\right)S+S^2\frac{1}{2}\frac{d^2P}{dS^2}\left(s\sigma\right)^2\right]dt$$
$$+\xi P\left[-r-m\left\{\frac{\left[r-(\mu+m\sigma)\right]}{s^2\sigma}\right\}dt\right]$$
$$+\xi\left[\left(\frac{\left[r-(\mu+m\sigma)\right]}{s^2\sigma}\right)\frac{\partial P}{\partial S}\left(s\sigma\right)Sdt\right]=0 \tag{12}$$

After rearranging terms and simplifying, this leads to:

$$\frac{\partial P}{\partial t}+S^2\frac{1}{2}\frac{d^2P}{dS^2}\left(s\sigma\right)^2+\left(r'-\varepsilon'\right)\frac{\partial P}{\partial S}S-r'P=0 \tag{12$'$}$$

where:

$$r'=r+m\frac{\left[r-(\mu+m\sigma)\right]}{s^2\sigma}\quad\text{and}\quad\varepsilon'=\delta-\frac{\left(m+s^2\sigma-s\sigma\right)\left[(\mu+m\sigma)-r\right]}{s^2\sigma}$$

Ambiguity impacts the fundamental valuation equation through the investor's discount rate, the Delta and Gamma of the option. r' and ε' are factors that distinguish ambiguity-based analysis from risk-neutral valuation with $m=2c-1$ and $s=\sqrt{4c\left(1-c\right)}$. r' is equivalent to a subjective (sometimes negative) investor discount rate, while yield ε' corresponds to a subjective investor value erosion factor dependent on Knightian or ambiguous beliefs. Eq. (12′) is a special case of Eq. (20) in Driouchi et al. (2015) and their exchange option pricing framework. Its MEU equivalent is also closely related to the equilibrium-based Eq. (17) of Faria and Correia-da-Silva (2012), confirming that the price of uncertainty plays an important role in contingent-claims valuation. When dW = dz, the analysis reduces to risk-neutral dynamics. In line with Merton (1973), a solution to pde (12′) with appropriate initial and terminal conditions for a European put option with strike price K and maturity T under ambiguity (following the standard Black–Scholes notation but noting the extended roles of r' and ε' in Eq. (13)) is:

$$P_0'=Ke^{-r'T}N\left(-\frac{\ln\left(\frac{S_0}{K}\right)+\left(r'-\varepsilon'-0.5\left(s\sigma\right)^2\right)T}{s\sigma\sqrt{T}}\right)$$
$$-S_0e^{-\varepsilon'T}N\left(-\frac{\ln\left(\frac{S_0}{K}\right)+\left(r'-\varepsilon'+0.5\left(s\sigma\right)^2\right)T}{s\sigma\sqrt{T}}\right)\quad(\forall c\ \in]0,\ 1[) \tag{13}$$

This solution corresponds to the set of possible prices satisfying Eq. (13) for $0<c<1$. K, S, T, r, σ and δ represent the usual inputs of constant volatility option pricing models under risk. N is the standard cumulative normal distribution function. In our extended framework, r' and ε' account for investors' heterogeneous beliefs and subjective discounting features proxying for their tendency to deviate from risk-neutrality under ambiguity. Behavioral variable s is a subjective volatility scalar which arises under ambiguity or when shocks and innovations are unexpected. As volatility is underestimated in this setting, both c and s should help better explain the IV–RV spread documented in the empirical literature. When $c=0.5$, $m=0$ and $s=1$, the above generalized put option formula reduces to the standard Black–Scholes solution for a European put option as $\varepsilon'=\delta$, $r'=r$ and $s\sigma=\sigma$.

Equation (13) is thus a natural extension of the Black–Scholes (risk-neutral) setup to the case of Choquet ambiguity (subsuming MEU where $m\neq 0$ and $s=1$). This implies that behavioral variables c, m and s contain subjective investor information that the standard risk-neutral formulation does not have. The aim of our paper is to quantify the effect of this subjective information on stock market volatility in the context of the 2006–2008 subprime crisis. We expect the ambiguity proxy c to contain predictive information beyond risk-neutral measures, and option implied ambiguity IC to be positively associated with S&P 500 index volatility in uncertain times. By back-solving Eq. (13) for σ numerically, given observed put option prices in abnormal market periods, ambiguity-based implied volatilities (IV_c) can be obtained for various levels of ambiguity attitudes or ignorance c (acting as determinant for m and s) allowing us to capture IV dynamics under changing heterogeneous beliefs (see Eq. (20) in Sect. 3.4). Moreover, one can infer market ambiguity parameters m, s and c implied from traded options prices if σ is known by minimizing the distance (i.e., absolute error) between observed market prices and model intrinsic values (e.g., the option price per day is characterized by a daily implied $c = P'^{-1}\left(K, S, T, r', \delta, \sigma\right))$ [see Eq. (21) in Sect. 3.4].

Based on Eq. (13), we posit that option implied ambiguity contributes to (understanding) fluctuations in the realized volatility of the underlying index and acts as a moderator in the lead–lag IV–RV relationship. We empirically test whether IC contains incremental information over IV in predicting RV, raising stock market volatility. This is explained by the fact that behavioral deviations from risk-neutrality (i.e., subjective sentiment) can be associated with positive changes in stock market volatility (De Bondt and Thaler 1987). Specifically, we assess the efficiency of the Choquet- and multiple-priors-based implied option information models and examine whether their ambiguity-based information is less biased than that of the risk-neutral case (implied by the standard Black–Scholes IV). For robustness purposes, we also test whether daily and weekly ambiguity IC has incremental information over IV in predicting RV using the stochastic volatility (SV) framework of Heston (1993). In line with standard literature on the information content of option implied volatility, OLS regressions are employed. Section 3 presents our data and the methodology we follow for the empirical analysis.

3 Data and methodology

3.1 Data

We study the subjective behavior of US index put option holders who invested in crash insurance, in the form of put options on the market index, in the period leading to the 2008 crash. Put option data (dividend-adjusted) on the S&P 500 index were obtained from Thomson Datastream. This covers daily settlement prices for the (European-type) put options (SPX), maturity dates (T) and strike prices (K). Daily settlement prices of the underlying stock market index were also collected for the calculation of daily realized volatilities (RV), option implied ambiguity (IC), ambiguity-based implied volatility (IV_c), and Black–Scholes and stochastic risk-neutral implied volatilities (BSIV and SV). In computing the option implied volatility and ambiguity measures, we focused our selection on contracts that matured on 18 Dec 2008,[9] the last trading day of our sample period of 2006–2008. This covers the periods preceding and surrounding the subprime crisis, the 2007–2008 credit crunch and the fall 2008 financial crash, all characterized by high levels of ignorance and ambiguity in global financial markets. Put option contracts that were out-of-the-money (moneyness K/S ranging from 0.51 to 0.93) for the longest periods were selected for further scrutiny as they

9 Our conclusions are unchanged if series of shorter maturity contracts are selected over the 2006–2008 period.

represent situations of investors' sustained uncertainty and downside risk expectations. This is in line with Bates (2008) who argues that OTM put options reflect investors' tendencies to insure themselves against crash risk. Implied volatilities IVs and implied ambiguity ICs are computed from daily settlement prices of the selected OTM put option contracts backed-out from our ambiguity-extended put option pricing model of Eq. (13). Yields of US T-bills and T-bonds over the maturity of each contract are used as the risk-free rate (r) in computing IV and IC for the index.

To examine the information efficiency of our ambiguity-based put option pricing framework of Eq. (13), we employ two regression designs (setups 1–2) in the analysis: (1) Setup 1 is aimed at estimating the IV–RV relationship under heterogeneous beliefs (under multiple c levels) revealing the moderating role of ambiguity factor c in this relationship and verifying whether such a relationship holds under different behaviors. (2) Setup 2 investigates the moderating effect of option implied ambiguity IC (with the c factor acting as a proxy for ambiguity) on the IV–RV relationship, focusing on estimating the incremental information content of this effect. Timelines and regression characteristics for each design are described in Sects. 3.2 and 3.3. We present Newey and West (1987) (NW) adjusted standard errors to overcome autocorrelation problems in the regression models. Our main purpose here is to compare the information content of option prices under alternative but comparable information frameworks (i.e., Choquet or multiple-priors vs. Black–Scholes), examining the effect of heterogeneous ambiguity beliefs and verifying whether the ambiguity-based option analysis provides more information than its risk-neutral counterpart.

3.2 Methodology: revealing the moderating role of ambiguity (setup 1)

This setup consists of estimating ambiguity-based implied volatilities IV_c (using a designated c value $0 < c < 1$ indicative of uncertainty preferences and ambiguity perceptions) over the 2006–2008 period. This is meant to illustrate the dynamics of IV under a range of ambiguity levels and establish whether the IV–RV association holds under pessimism and/or optimism. It is analogous to verifying whether the IV–RV relationship is represented by multiple lines with different slopes. This helps illustrate that the risk-neutral IV–RV relationship is not unique and may actually miss alternative ambiguity behaviors. For each contract, a series of (17) ambiguity-based implied volatilities with c values ranging from 0.1 to 0.9 (in 0.05 increments) were obtained for regression purposes. Recall that $c < 0.5$ represents pessimism, $c = 0.5$ risk-neutrality, and $c > 0.5$ optimism. Serving as a preamble to the findings of setup 2, setup 1 should (dis)prove the (superiority) lack of uniqueness of the risk-neutral IV–RV relationship and underline the role of ambiguity in this relationship.

In line with prior research in this area (e.g., Fung 2007; Taylor et al. 2010), we use the following lead–lag regression model on our OTM put option price series for setup 1:

$$RV_{t,T} = \beta_0 + \beta_1 IV_{c,t} + \varepsilon \tag{14}$$

where $RV_{t,T}$ represents the ex-post realized index volatility from t to T (where T is the maturity date of the option and T > t) and $IV_{c,t}$ represents ambiguity-based option implied volatility under ignorance level c. Findings for this model are presented in Sect. 4, Table 3. The corresponding timeline is shown in Table 1. To verify the robustness of the information content of index option prices under ambiguity, different sample horizons are used. The regression schedule for these different horizon lengths is shown in Table 1. This timeline is also valid for setup 2.

Table 1 Regression timeline for setups 1 and 2

Regression length (months)	Start day of data series	End day of data series
12	24/12/2007	20/11/2008
15	24/09/2007	
18	18/06/2007	
21	19/03/2007	
24	18/12/2006	
27	18/09/2006	

The table shows the regression schedule followed for setups 1–2 for different period lengths during 2006–2008. Data coverage: 09/2006–12/2008

3.3 Methodology: estimating the moderating effect of implied ambiguity (setup 2)

This setup consists of extracting ambiguity aversion (implied c after inverting Eq. (13)) from put option prices and estimating its moderating effect on the IV–RV relationship. IC is estimated by minimizing the distance (absolute error) between model option prices and observed index option prices (see Eq. 21). Regressions analyzing the information content of IV under the standard Black–Scholes model (as benchmark) compared to our ambiguity-adjusted model of Eq. (13) are carried out for each option contract. Regressions with interaction terms for ambiguity are implemented daily and weekly to test whether option implied ambiguity IC contains incremental information (beyond risk-neutral IV) regarding RV. Risk-neutral IVs are computed along the corresponding IC value on that day. The interaction term BSIV × IC, which we refer to as ambiguity-adjusted implied volatility ICBSIV, is used as an explanatory factor in the statistical regressions. To quantify the moderating effect of investors' implied ambiguity IC in the IV–RV linkage, the following lead–lag setup 2 model is implemented:

$$RV_{t,T} = \beta_0 + \beta_1 BSIV_t + \beta_2 BSIV_t \times IC_t + \varepsilon \tag{15}$$

where $RV_{t,T}$ is the realized index volatility as in Eq. (14), $BSIV_t$ stands for the Black–Scholes implied volatility (BSIV as defined earlier), and IC_t represents option implied ambiguity from observed index put option prices. Equation (15) is suitable for estimating the interaction effect of ambiguity on the IV–RV relationship. Equation (15) is then compared to the (risk-neutral) Black–Scholes IV information structure. We use the following benchmark (lead–lag) model for comparison in setup 2:

$$RV_{t,T} = \beta_0 + \beta_1 BSIV_t + \varepsilon \tag{16}$$

where $RV_{t,T}$ and $BSIV_t$ are defined as before. Despite several drawbacks, the BS setting remains a reasonable benchmark for comparison and exhibits accurate forecasting power over alternative information frameworks (e.g., Clark and Gosh 2004; Muzzioli 2010; Ayadi et al. 2014). Our aim here is to compare fundamental frameworks that are economically comparable in terms of information content and predictive power and that enable us to isolate the ambiguity aversion effects induced by the IC factor. The standard BS framework is a plausible choice as a benchmark model for comparison. For robustness, we also compare a variant of Eq. (15) under stochastic volatility based on the Heston (1993) formulation using SV in lieu of BSIV with the equivalent of Eq. (16) based on risk-neutral SV.

The two regression models described in Eqs. (15) and (16) are implemented on the relevant OTM index option contracts. If the ambiguity-adjusted IV formulation provides more information content than the risk-neutral benchmark model, regressions based on Eq. (15) should yield a higher R^2 and more significant coefficients than those of Eq. (16).

Additionally, in Sect. 4.3, we test whether Choquet-based implied volatility (s × BSIV), where $s = (4c(1-c))^{1/2}$ is the ambiguity-driven volatility scalar from Eq. (13), is a superior predictor of RV compared to BSIV. As in setup 1, different sample lengths are used to examine the robustness of findings for the 2006–2008 period. In brief, setup 2 measures the moderating effect of option implied ambiguity IC in the IV–RV linkage. Setup 1 verifies whether moderation exists by examining the multiplicity of the IV–RV relationship under different c levels. The independent incremental information content of IC is also presented as additional findings in Sect. 4.3 where the following lead–lag information structure is tested and compared to Eq. (16):

$$RV_{t,T} = \beta_0 + \beta_1 BSIV_t + \beta_2 IC_t + \varepsilon \tag{17}$$

where $RV_{t,T}$, $BSIV_t$ and IC_t are defined as before. Findings for this model are presented in Sect. 4, Table 5.

3.4 Variables specification

3.4.1 Realized volatility (RV)

This is computed by taking the ex-post annualized sample standard deviation (SD) of the daily index returns over a specific period. For example, the RV on day t in the sample series is measured by the sample SD of daily returns from day t to T, where day T is the last day of the sample series, defined as the maturity of the corresponding option contract. This follows Christensen and Prabhala (1998) and avoids the maturity mismatch problem. Our time series stop at T-22 to ensure all realized volatility figures are computed with a minimum of 22 observations. A standard number of 252 trading days was adopted for annualization. The realized volatility is thus based on the following:

$$RV_{t,T} = \sqrt{\frac{\sum_{i=t+1}^{T} \left(R_i - \overline{R_{t,T}}\right)^2}{n-1}} \times \sqrt{252} \tag{18}$$

where $RV_{t,T}$ is the ex-post realized volatility from day t to T (where T is the maturity date of the option and T > t) and R_i is the daily index return on day i. $\overline{R_{t,T}}$ is the mean daily index return from day t to T, and n is the sample size which is equivalent to $(T-t)$.

3.4.2 Black–Scholes implied volatility (BSIV)

The standard Black and Scholes (1973) model is employed for estimating the benchmark (risk-neutral) implied volatility:

$$BSIV_t = J\left(K, S_t, (T-t), r_t, \delta\right) \tag{19}$$

where J stands for the inverse of the risk-neutral Black and Scholes (1973) put option function.

3.4.3 Ambiguity-based implied volatility (IV_c)

This is obtained after setting the c parameter to a specific level of ambiguity aversion and backing-out IV_c under heterogeneous ambiguity beliefs.

$$IV_{c,t} = L\left(K, S_t, (T-t), r_t, \delta, c_t\right) \tag{20}$$

where L stands for the inverse of the Choquet-based put option function (and its multiple-priors equivalent) based on Eq. (13).

3.4.4 Option implied ambiguity (IC)

Ambiguity-adjusted implied volatility ICBSIV is obtained by multiplying the standard BSIV with the implied IC factor. Ambiguity proxy IC is backed-out from Eq. (13) by minimizing the absolute error between observed index options prices and model intrinsic values such that:

$$AE_t^{IC} = \min_{c|0<c<1}\left\{\left|P_t^P\left(S_t, K, r_t, (T-t), \sigma_t, \mu_t, c_t\right) - P_t^M\right|\right\} \tag{21}$$

where AE_t^{IC} stands for the absolute error between observed index options prices and model intrinsic values, function P_t^P stands for the Choquet-based put option price (Eq. 13) (and its multiple-priors equivalent) derived in Sect. 2. P_t^M is the observed index option price, S, K and r are as defined before, $T-t$ stands for the time to maturity in units of a year, and μ_t is the subjective expected return. The extraction of option implied ambiguity IC reduces to a simple minimization problem under Eq. (21). We employ the annualized average daily return over the previous 2 years as an approximation for μ_t.

3.4.5 Stochastic volatility (SV)

As a robustness check, we compare the predictive power of option implied ambiguity IC to stochastic implied volatility based on Heston's (1993) stochastic volatility specification. Since the calibration of the Heston model is not the main theme of this paper and does not affect the role of SV as a benchmark predictor, we compute the Heston SV according to the standard parameters stated below. The Heston SV is defined as:

$$SV_t = H\left(K, S_t, (T-t), r_t, \rho, \lambda, \kappa, \theta, \sigma_V\right) \tag{22}$$

where H stands for the inverse of the Heston pricing function, K, S_t, $(T-t)$ and r_t are as defined before, ρ is the correlation between the two Wiener processes implicit in the Heston model, λ is the price of volatility risk, κ is the mean reversion parameter, θ is the long-run mean of variance, and σ_V is the volatility of volatility. We define these model parameters as $\rho = -0.25$, $\lambda = 0$, $\kappa = 1$, $\theta = 0.03$, $\sigma_V = 0.5$ over the entire 2006–2008 period.

4 Empirical findings

This section presents our empirical findings, along with descriptive statistics and graphical representations of investors' option implied ambiguity. Figure 1 illustrates the subjective behavior of put option holding investors during the 2006–2008 period. It plots the dynamics of implied volatility for the index under pessimism ($c = 0.2$), risk-neutrality ($c = 0.5$) and optimism ($c = 0.8$). It also shows the S&P 500 index level for comparison. A consistent

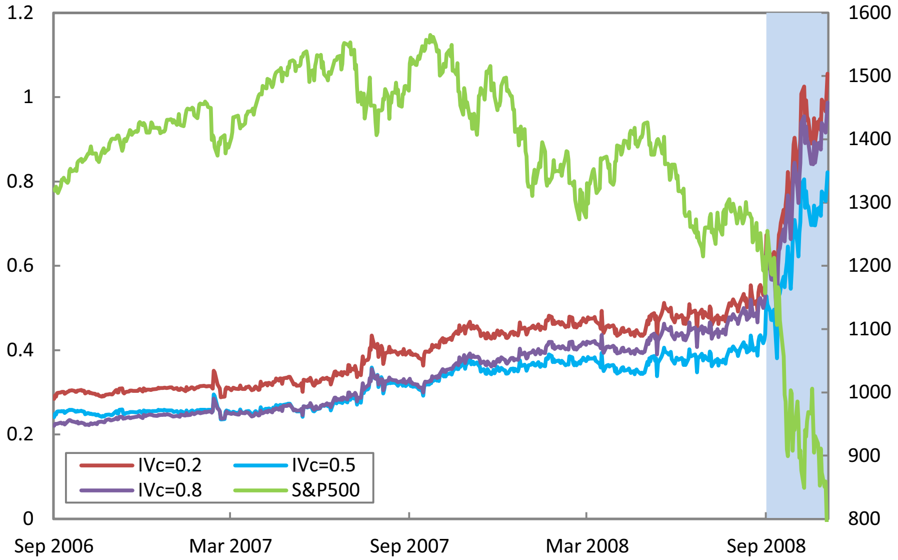

Fig. 1 Ambiguity-based implied volatilities (IV_c) versus daily index levels (2006–2008)

pattern emerges involving gradual increases in all three implied volatilities in the period leading up to the credit crunch and the September 2008 crash when volatilities suddenly jump upwards as a result of the Lehman Brothers collapse. The figure suggests that after September 2007, risk-neutral implied volatility ($IV_{0.5}$) was likely to underestimate put option value relative to subjective ambiguity sentiments, confirming that risk-neutral analysis may understate the probability of rare adverse events (e.g., occurrence of a market crash or extreme downside losses in index levels). For put options to be more valuable and in-the-money, investors needed to bet against or short the index. Unlike their pessimistic or optimistic counterparts, risk-neutral investors seem to underestimate the likelihood of such an adverse event. This suggests that ambiguity aversion may have an impact on the dispersion of index returns and moderate the IV–RV relationship in uncertain times. Figure 1 also shows how the upward trends in implied volatilities tend to correlate with downside fluctuations in underlying stock index levels, confirming an inverse association between put volatilities and the index. The shaded area to the right highlights the unusual dynamics of implied volatilities surrounding the fall 2008 global banking crash.

Figure 2 depicts option implied ambiguity IC versus risk-neutral BSIV and index levels, documenting clear patterns of ambiguity aversion ($c < 0.5$) during the subprime crisis and shifts in ambiguity from 2006 to 2008. The IC indicator increases incrementally (i.e., from high to low ambiguity aversion) in the period leading to the September 2008 crash. This can be explained by improved option moneyness and uncertainty resolution around the options' maturity dates (i.e., investors' optimism about their put option prospects). This finding rejects the hypothesis of risk-neutrality among investors.

Table 2 reports summary statistics of ambiguity-based implied volatilities IV_c and implied ambiguity IC for US SPX index options over 2006–2008 under multiple-priors and Choquet ambiguity. MEU ambiguity modeling involves uncertainty in the drift of the stochastic process of Eq. (1) by entertaining different values for parameter m (assuming $s = 1$), whereas CEU involves uncertainty in both the drift (m) and the volatility scalar s ($s \neq 1$). We assess implied

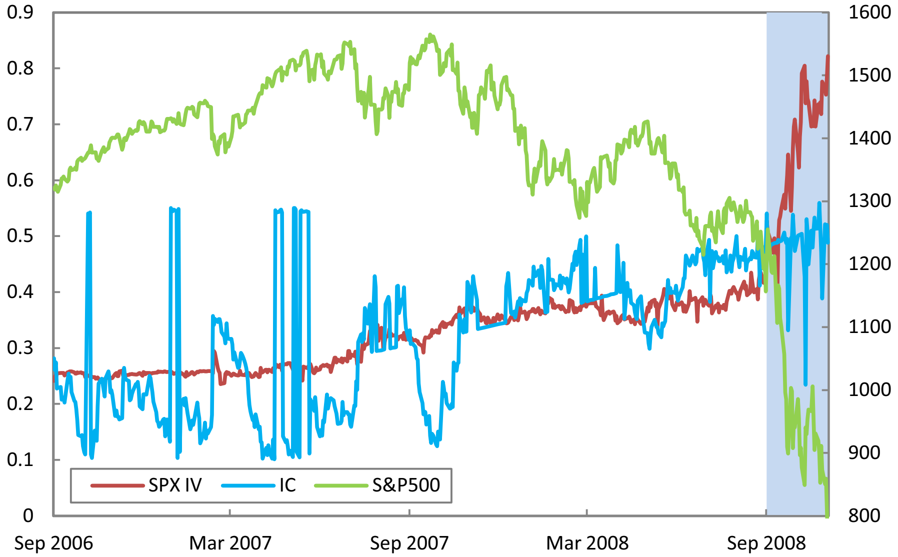

Fig. 2 Option implied ambiguity (IC) versus Black–Scholes implied volatility (SPXIV) and daily index levels (2006–2008)

volatility behaviors under pessimism ($c = 0.2$), optimism ($c = 0.8$) and risk-neutrality ($c = 0.5$) using designated c values (Eq. 20). IC dynamics (see Eq. 21) are also presented in the table. Findings show that ambiguity-based IV_c estimates are higher than BSIV for CEU in line with the logic of probability weighting, but not for MEU in accord with the maxmin criterion. IC descriptive statistics confirm that ambiguity aversion is consistently observed throughout the 2006–2008 period. Sections 4.1 and 4.2 summarize our regression findings for setups 1 and 2.

4.1 Implied volatility under heterogeneous ambiguity beliefs (setup 1)

Equation (14) predicts that the IV–RV relationship holds under different c levels (i.e., beyond risk-neutrality) such that IV_c is a significant and efficient predictor of RV. Table 3 presents our univariate regression results. We report results for two sets of ambiguity aversion ($c = 0.2$ and $c = 0.4$) and two sets of ambiguity-seeking behaviors ($c = 0.6$ and $c = 0.8$) under both MEU and CEU. The benchmark case based on $c = 0.5$ is also shown in the table. The t-statistics are Newey and West (1987) adjusted. The results confirm that the information content of option implied volatility holds under pessimism and optimism (i.e., it is not exclusive to risk-neutral analytics) for both MEU and CEU. The IV–RV relationship is significant across a range of ambiguity levels including high pessimism or high optimism, confirming the moderating behavior (quantified in setup 2) from c-ignorance and the existence of multiple IV–RV lines with different slopes. More importantly, we find that our subjective IVs ($c \neq 0.5$) tend to outperform BSIV in terms of information content and predictive power across all sub-periods examined, disproving the superiority of the risk-neutral IV–RV association. The MEU specification seems to perform better under pessimism, while the CEU formulation shows better goodness of fit under optimism in line with decision theory conjectures. These results confirm that standard (risk-neutral) options analysis can be incomplete. Relying on risk-neutral implied volatility for RV prediction therefore ignores investor sentiment towards

Table 2 Descriptive statistics for US implied volatilities and ambiguity

Variable	US SPX options (MEU)					US SPX options (CEU)				
	Min	Max	Mean	Median	SD	Min	Max	Mean	Median	SD
1. $IV_{c=0.2}$ (pessimism)	0.200	0.797	0.331	0.326	0.109	0.200	1.055	0.427	0.418	0.144
2. $IV_{c=0.5}$ (risk−neutrality)	0.236	0.822	0.344	0.339	0.109	0.236	0.822	0.344	0.339	0.109
3. $IV_{c=0.8}$ (optimism)	0.211	0.815	0.326	0.314	0.117	0.221	0.987	0.371	0.348	0.150
4. IC	0.103	0.570	0.338	0.354	0.127	0.102	0.560	0.330	0.344	0.126

The table reports summary statistics of ambiguity-based implied volatilities (IV_c) and implied ambiguity (IC) for US SPX index options over 2006–2008 under CEU and MEU. IC is obtained by minimizing the distance between model prices and observed option prices under each ambiguity specification

Table 3 Implied volatility information under heterogeneous ambiguity beliefs

	SPX IV_c results (MEU)						SPX IV_c results (CEU)					
	12 months	15 months	18 months	21 months	24 months	27 months	12 months	15 months	18 months	21 months	24 months	27 months
Pessimism												
$c = 0.2$												
Cst	0.296	0.226	0.175	0.149	0.128	0.109	0.303	0.234	0.183	0.157	0.136	0.117
	(8.617)	(5.880)	(4.722)	(4.438)	(4.048)	(3.613)	(9.092)	(6.306)	(5.139)	(4.880)	(4.443)	(3.985)
IV	0.665	0.699	0.753	0.797	0.826	0.845	0.657	0.695	0.749	0.794	0.822	0.842
	(7.310)	(7.181)	(7.660)	(8.356)	(8.841)	(9.206)	(7.394)	(7.272)	(7.746)	(8.436)	(8.876)	(9.215)
R^2 (%)	44.21	48.88	56.71	63.58	68.17	71.44	43.13	48.26	56.16	63.10	67.63	70.90
F	187.81	288.79	487.39	763.01	1074.99	1418.00	179.76	281.72	476.45	747.37	1048.93	1381.68
$c = 0.4$												
Cst	0.300	0.229	0.175	0.147	0.123	0.102	0.301	0.229	0.176	0.148	0.124	0.103
	(8.857)	(6.047)	(4.763)	(4.392)	(3.822)	(3.218)	(8.895)	(6.082)	(4.801)	(4.434)	(3.860)	(3.254)
IV	0.644	0.678	0.734	0.782	0.810	0.830	0.643	0.678	0.734	0.782	0.811	0.830
	(7.453)	(7.346)	(7.800)	(8.476)	(8.857)	(9.107)	(7.458)	(7.351)	(7.806)	(8.482)	(8.859)	(9.107)
R^2 (%)	41.43	45.95	53.84	61.10	65.69	68.85	41.40	45.98	53.86	61.12	65.69	68.84
F	167.61	256.77	433.88	686.34	960.98	1253.04	167.45	257.02	434.28	686.92	961.20	1252.81
Optimism												
$c = 0.6$												
Cst	0.302	0.231	0.179	0.152	0.129	0.109	0.302	0.232	0.180	0.153	0.130	0.110
	(9.013)	(6.203)	(4.980)	(4.695)	(4.139)	(3.558)	(9.047)	(6.239)	(5.028)	(4.758)	(4.204)	(3.628)
IV	0.645	0.683	0.739	0.787	0.815	0.834	0.646	0.685	0.740	0.788	0.816	0.835
	(7.471)	(7.362)	(7.840)	(8.551)	(8.931)	(9.196)	(7.470)	(7.360)	(7.841)	(8.554)	(8.934)	(9.202)
R^2 (%)	41.62	46.67	54.63	61.86	66.35	69.51	41.75	46.87	54.83	62.04	66.51	69.65
F	168.94	264.24	447.91	708.74	989.88	1292.49	169.85	266.45	451.59	714.15	996.77	1301.32

Table 3 continued

	SPX IV_c results (MEU)						SPX IV_c results (CEU)					
	12 months	15 months	18 months	21 months	24 months	27 months	12 months	15 months	18 months	21 months	24 months	27 months
c = 0.8												
Cst	0.302	0.234	0.187	0.165	0.145	0.128	0.307	0.246	0.206	0.188	0.171	0.158
	(9.104)	(6.408)	(5.425)	(5.389)	(5.039)	(4.690)	(9.656)	(7.211)	(6.580)	(6.885)	(6.762)	(6.689)
IV	0.668	0.712	0.766	0.809	0.835	0.854	0.701	0.750	0.799	0.836	0.858	0.875
	(7.372)	(7.259)	(7.805)	(8.587)	(9.059)	(9.457)	(7.291)	(7.229)	(7.857)	(8.683)	(9.202)	(9.688)
R^2 (%)	44.67	50.72	58.69	65.45	69.75	72.94	49.20	56.27	63.85	69.87	73.67	76.56
F	191.36	310.76	528.41	827.98	1157.76	1528.53	229.53	388.64	657.02	1013.34	1404.42	1852.42
Risk neutrality												
c = 0.5												
Cst	0.301	0.230	0.177	0.149	0.125	0.103	0.301	0.230	0.177	0.149	0.125	0.103
	(8.950)	(6.129)	(4.852)	(4.500)	(3.911)	(3.293)	(8.950)	(6.129)	(4.852)	(4.500)	(3.911)	(3.293)
IV	0.641	0.677	0.733	0.781	0.810	0.829	0.641	0.677	0.733	0.781	0.810	0.829
	(7.480)	(7.375)	(7.836)	(8.522)	(8.889)	(9.129)	(7.480)	(7.375)	(7.836)	(8.522)	(8.889)	(9.129)
R^2 (%)	41.13	45.83	53.75	61.05	65.60	68.73	41.13	45.83	53.75	61.05	65.60	68.73
F	165.58	255.52	432.33	684.95	957.14	1246.33	165.58	255.52	432.33	684.95	957.14	1246.33

The table summarizes our IV_c regression results for the univariate models (setup 1: Revealing the moderating role of ambiguity perceptions) described in Eq. (14). We present MEU and CEU results for two sets of ambiguity aversion (i.e., c = 0.2 and c = 0.4) and two sets of ambiguity-seeking behaviors (c = 0.6 and c = 0.8). This setup is equivalent to identifying multiple lines with different slopes after setting variable c to a specific level within the]0, 1[interval (NW t-statistics in parentheses) ambiguity and does not capture fully the dispersion of index returns in uncertain times. Our hypothesis on the multiplicity of the IV–RV relationship is thus validated.

As a follow-up to setup 1, Table 4 compares the predictive performance of BSIV × IC versus BSIV to ascertain the incremental information content of option implied ambiguity IC. It shows results relating to the estimation of the moderating effect of IC on the IV–RV linkage under both MEU and CEU. Equation (15) predicts that the ambiguity proxy IC contains predictive RV information beyond BSIV and that IC moderates the relationship between IV and RV such that IV will lead to even higher RV.

4.2 The information content of ambiguity-adjusted implied volatility (setup 2)

In comparing the predictive power of ICBSIV (under both MEU and CEU) vs. that of BSIV over various prediction horizons (setup 2), Table 4 regressions show that ICBSIV-related estimates are less biased than the standard BSIV benchmark counterparts. ICBSIV coefficients are significantly different from zero and coefficient significance and predictive power are stronger (i.e., nearer to unity) when accounting for ambiguity. R^2 is consistently higher when including BSIV × IC interactions in the regressions. An interesting finding is that BSIV becomes insignificant during the 06/2007–12/2008 period once ICBSIV is introduced in the regressions. This suggests that ambiguity-adjusted implied volatility subsumes information from its risk-neutral counterpart during highly uncertain or ambiguous events (i.e., the credit crunch and the fall 2008 crash). The ICBSIV estimate is, therefore, a better predictor than BSIV for this time window. The positive moderating/interaction effect of IC is validated for the entire 2006–2008 period (VIFs < 3.2). The incremental information content of our ambiguity-based indicators is robust, sometimes reaching differences in R^2 of up to 11 *pp* and 13 *pp* for MEU and CEU, respectively. While the MEU specification outperforms BSIV across all sub-periods, we find that the CEU framework captures more predictive RV information than its MEU counterpart. This suggests that probability weighting might be more common than simple maxmin criteria in option trading decisions. This holds across the entire sample period and over various time windows. It thus validates our hypothesis that ambiguity-adjusted volatility ICBSIV is more efficient, less biased and contains extra information relative to BSIV. This conclusion holds after controlling for residuals' bias via orthogonal regressions.

The overall results confirm the positive moderating role of option implied ambiguity IC in the IV–RV relationship, with IC containing incremental information. This complements the findings from setup 1 by corroborating the moderating role of investors' ambiguity aversion in the IV–RV relationship. We estimate the degree of moderation of the c variable implied by observed put option prices around the subprime crisis. The positive sign of the BSIV × IC coefficients indicates that ambiguity aversion, as implied from put options prices, leads to higher realized index volatility in times of uncertainty. In other words, ambiguity captured in index put option prices is associated with higher volatility in the stock market. These results hold using daily and weekly data.

4.3 Additional results and robustness tests

For robustness, we ran additional tests of information content. First, we assessed the informational efficiency of the IC factor both in itself and in conjunction with BSIV, by regressing IC and BSIV on RV over the 2006–2008 period under multiple-priors and Choquet ambiguity. Table 5 summarizes our results. Findings show that option implied ambiguity is significant and contains incremental information over BSIV. Forecasts are significantly improved when

Table 4 Regression results: the information content of ambiguity-adjusted implied volatility versus risk-neutral implied volatility

	SPX-BSIV				SPX-BSIV and ICBSIV (MEU)					SPX-BSIV and ICBSIV (CEU)				
	Cst	BSIV	F	Adj R^2 (%)	Cst	BSIV	BSIV × IC	F	Adj R^2 (%)	Cst	BSIV	BSIV × IC	F	Adj R^2 (%)
Mar 2008 to Dec 2008	0.396	0.618	106.01	37.77	0.422	–	0.683	150.26	46.32	0.428	–	0.688	154.83	47.07
	(11.858)	(6.243)			(16.131)	–	(6.428)			(17.117)	–	(6.531)		
Dec 2007 to Dec 2008	0.301	0.641	165.58	40.88	0.347	–	0.682	205.57	46.22	0.354	–	0.689	214.47	47.28
	(8.950)	(7.480)			(13.659)	–	(7.741)			(14.323)	–	(7.736)		
Sep 2007 to Dec 2008	0.230	0.677	255.52	45.65	0.310	–	0.757	405.60	57.18	0.318	–	0.764	424.17	58.27
	(6.129)	(7.375)			(15.267)	–	(8.973)			(16.323)	–	(8.978)		
Jun 2007 to Dec 2008	0.177	0.733	432.33	53.63	0.286	–	0.798	650.30	63.51	0.294	–	0.804	681.41	64.59
	(4.852)	(7.836)			(17.442)	–	(10.28)			(18.431)	–	(10.15)		
Mar 2007 to Dec 2008	0.149	0.781	684.95	60.96	0.208	0.351	0.479	412.87	65.29	0.284	–	0.816	871.63	66.53
	(4.500)	(8.522)			(5.814)	(2.597)	(3.875)			(23.172)	–	(11.70)		
Dec 2006 to Dec 2008	0.125	0.810	957.14	65.53	0.182	0.420	0.435	564.32	69.13	0.212	0.297	0.557	595.19	70.26
	(3.911)	(8.889)			(5.341)	(3.343)	(3.979)			(5.838)	(2.124)	(4.258)		
Sep 2006 to Dec 2008	0.103	0.829	1246.33	68.68	0.162	0.458	0.412	726.84	71.88	0.191	0.335	0.536	763.77	72.87
	(3.293)	(9.129)			(4.838)	(3.793)	(4.053)			(5.446)	(2.512)	(4.385)		

NW t-statistics in parentheses

The table compares the predictive performance of BSIV × IC (ICBSIV) and BSIV to ascertain the incremental information content of the implied c factor (IC) from Eqs. (13) and (15). It presents MEU and CEU results relating to the estimation of the moderating/interaction effect of option implied ambiguity IC on the RV-IV linkage (setup 2: Estimating the moderating effect of implied ambiguity). IC is obtained after minimizing the distance between observed options prices and model intrinsic prices

Table 5 The incremental information content of option implied ambiguity

Ambiguity framework	Independent variables				
	Constant	IC	BSIV	CBOE SKEW index	Adj R^2 (%)
1. SPX (MEU)	0.237 (10.37)	0.569 (6.18)	–	–	32.23
2. SPX (MEU)	0.103 (3.29)	–	0.829 (9.13)	–	68.68
3. SPX (MEU)	0.072 (2.80)	0.222 (3.95)	0.722 (8.17)	–	72.44
4. SPX (MEU)	0.850 (5.42)	–	0.748 (11.65)	−0.258 (−4.94)	74.66
5. SPX (MEU)	0.723 (5.39)	0.166 (4.26)	0.680 (10.80)	−0.222 (−4.87)	76.64
6. SPX (CEU)	0.209 (9.50)	0.672 (7.80)	–	–	45.00
7. SPX (CEU)	0.103 (3.29)	–	0.829 (9.13)	–	68.68
8. SPX (CEU)	0.077 (3.23)	0.283 (4.25)	0.663 (7.52)	–	73.92
9. SPX (CEU)	0.850 (5.42)	–	0.748 (11.65)	−0.258 (−4.94)	74.66
10. SPX (CEU)	0.669 (5.19)	0.210 (4.45)	0.643 (10.27)	−0.203 (−4.62)	77.24

NW t-statistics in parentheses

The table summarizes our regression results for US SPX options under MEU and CEU. We assess the informational efficiency of the IC factor (where implied c is extracted from option prices based on Eqs. (13), (17) and (21)) both by itself and in conjunction with BSIV by regressing BSIV and IC on RV over the whole 2006–2008 period such that: RV = f (BSIV, IC). These results are also valid for shorter time windows within 2006–2008

taking the combined effects of BSIV and IC in the MEU and CEU regressions (VIFs < 2). These results hold even after controlling for fat tail risk (proxied by the CBOE SKEW index) (Table 5) and volatility of volatility effects (proxied by the CBOE VVIX) in the regression models (unreported).[10] This confirms that our ambiguity proxy IC has incremental information content over BSIV and other implied risk-neutral option information. Once again, we find that option implied ambiguity is positively associated with stock market volatility over the 2006–2008 period. This is valid for both daily and weekly data frequencies.

As additional robustness, we tested the extent to which IC moderates (risk-neutral) IV under a stochastic volatility SV specification, relaxing the assumption of constant volatility characterizing the Black–Scholes model. Our findings are robust (daily and weekly) to this alternative specification with IC containing extra information beyond the Heston (1993) SV, both on its own (IC) and in interaction (SV $\times$ IC). These results are presented in Table 6 (VIFs < 2 in the bivariate regressions). Our findings are also valid when the VIX index, historical or GARCH volatility are employed (instead of BSIV or SV) or when using the Investor Intelligence Index from Investors Intelligence (II) as an alternative proxy for c in the regressions. Despite the relatively small time window considered, out-of-sample forecasts of realized volatility confirm that our MEU and CEU-based indicators outperform risk-neutral BS forecasts. Our findings once again hold after controlling for residuals' bias via orthogonal regressions.

Finally, we assessed the informational efficiency of the s $\times$ BSIV factor from Eq. (13) compared to BSIV by running univariate RV regressions over the 2006–2008 period under CEU. Table 7 summarizes these results, confirming that s $\times$ BSIV is a superior predictor of RV than BSIV across all sub-periods. We also find that the well-known volatility smile becomes flatter when accounting for ambiguity in option pricing (Fig. 3). Our overall results

[10] The significance of the IC variable is also maintained after controlling for realized skewness and kurtosis in the regressions.

Table 6 Regression results: the information content of ambiguity-adjusted implied volatility and option implied ambiguity vs. risk-neutral stochastic volatility

	SPX-SV				SPX-SV and SVIC					SV + IC				
	Cst	SV	F	Adj R^2 (%)	Cst	SV	SV × IC	F	Adj R^2 (%)	Cst	SV	IC	F	Adj R^2
Mar 2008 to Dec 2008	0.439	0.630	113.21	39.34	0.459	–	0.680	147.99	45.94	0.164	0.367	0.508	121.26	58.16
	(17.030)	(6.585)			(22.019)	–	(6.702)			(2.660)	(3.872)	(4.616)		
Dec 2007 to Dec 2008	0.365	0.612	142.04	37.21	0.395	–	0.658	180.48	42.99	0.090	0.397	0.416	118.81	49.75
	(14.632)	(8.209)			(19.604)	–	(8.200)			(1.252)	(4.926)	(3.964)		
Sep 2007 to Dec 2008	0.308	0.655	226.82	42.70	0.356	–	0.731	346.81	53.30	0.151	0.398	0.475	215.52	58.61
	(11.980)	(8.368)			(21.181)	–	(9.010)			(4.517)	(5.978)	(5.380)		
Jun 2007 to Dec 2008	0.276	0.723	408.38	52.20	0.332	–	0.780	577.59	60.72	0.166	0.437	0.442	324.26	63.41
	(14.210)	(10.040)			(25.427)	–	(10.21)			(6.754)	(6.765)	(5.311)		
Mar 2007 to Dec 2008	0.273	0.782	685.91	60.99	0.322	–	0.819	891.44	67.03	0.210	0.615	0.284	430.25	66.22
	(21.706)	(12.406)			(32.975)	–	(11.44)			(12.743)	(9.402)	(3.928)		
Dec 2006 to Dec 2008	0.260	0.812	973.51	65.91	0.311	–	0.840	1204.4	70.52	0.209	0.661	0.253	587.00	69.97
	(23.386)	(12.908)			(36.446)	–	(11.96)			(15.173)	(10.594)	(3.954)		
Sep 2006 to Dec 2008	0.245	0.827	1225.39	68.31	0.299	–	0.852	1705.8	72.62	0.196	0.666	0.261	748.92	72.48
	(22.019)	(12.933)			(37.344)	–	(12.19)			(14.641)	(10.991)	(4.270)		

NW t-statistics in parentheses

The table compares the predictive performance of SV × IC (SVIC) and Heston SV to ascertain the incremental information content of the implied c factor (IC) from Eqs. (13) and (15). It presents results relating to the estimation of the effect of option implied ambiguity (CEU-based IC) on the RV-SV linkage (setup 2: Estimating the moderating effect of implied ambiguity). We also assess the informational efficiency of the implied IC factor (where implied c is extracted from option prices based on Eqs.(13) and (21)) in conjunction with Heston SV by regressing SV and IC on RV over the 2006–2008 period such that: RV = f (SV, IC). IC is obtained after minimizing the distance between observed options prices and model intrinsic prices

Table 7 The information content of Choquet-based implied volatility versus Black–Scholes implied volatility

	SPX-BSIV				SPX-s × BSIV			
	Cst	IV	F	Adj R^2 (%)	Cst	s × IV	F	Adj R^2 (%)
Mar 2008 to Dec 2008	0.396	0.618	106.01	37.77	0.394	0.637	117.47	40.23
	(11.858)	(6.243)			(12.074)	(6.302)		
Dec 2007 to Dec 2008	0.301	0.641	165.58	40.88	0.302	0.655	178.15	42.67
	(8.950)	(7.480)			(9.177)	(7.522)		
Sep 2007 to Dec 2008	0.230	0.677	255.52	45.65	0.242	0.720	325.28	51.70
	(6.129)	(7.375)			(7.590)	(8.006)		
Jun 2007 to Dec 2008	0.177	0.733	432.33	53.63	0.204	0.771	546.16	59.38
	(4.852)	(7.836)			(7.376)	(9.010)		
Mar 2007 to Dec 2008	0.149	0.781	684.95	60.96	0.192	0.811	842.32	65.76
	(4.500)	(8.522)			(8.678)	(10.394)		
Dec 2006 to Dec 2008	0.125	0.810	957.14	65.53	0.176	0.836	1164.99	69.83
	(3.911)	(8.889)			(8.691)	(11.029)		
Sep 2006 to Dec 2008	0.103	0.829	1246.33	68.68	0.161	0.853	1511.25	72.67
	(3.293)	(9.129)			(8.299)	(11.420)		

NW t-statistics in parentheses

The table compares the predictive power of s × BSIV as a predictor of RV relative to BSIV to ascertain the incremental information content of the implied c factor (IC) from Eqs. (13) and (21). IC is obtained after minimizing the distance between observed options prices and model intrinsic prices. $s = (4ic(1 - ic))^{1/2}$

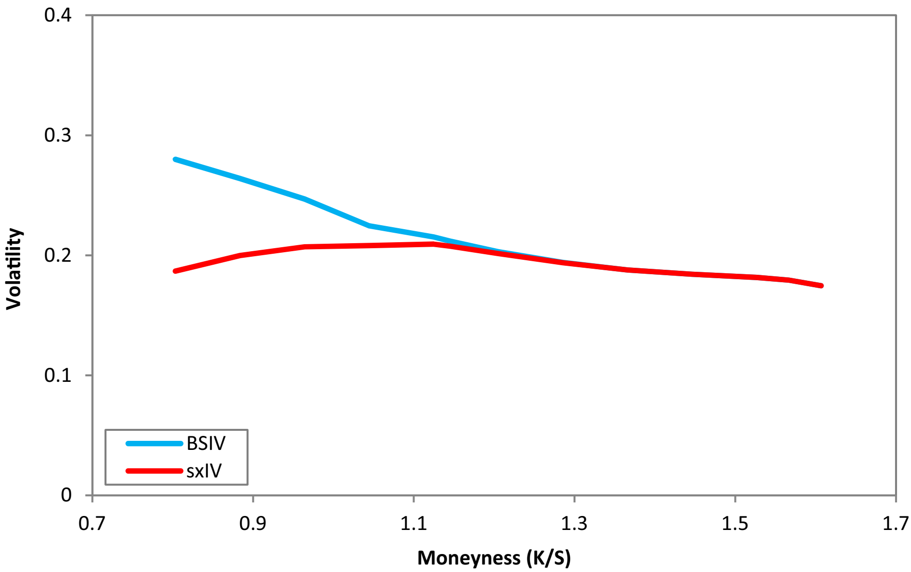

Fig. 3 Ambiguity-based volatility smile (s × IV) versus Black–Scholes volatility smile (BSIV)

hold using both daily and weekly data specifications. The conclusions are also unchanged to how IV and IC are extracted from observed market prices (i.e., separately or simultaneously), moneyness specification or how RV and μ are estimated. For comparative purposes, we also ran regressions on 2004–2006 SPX put options data (non OTM) to test how well IC moderates the IV–RV relationship in less ambiguity-intense periods. Although the moderating role of ambiguity is still present and significant, its impact and predictive power are weaker during normal periods involving lower ambiguity. As there were less frequent deviations from risk-neutrality among option investors during the 2004–2006 period, this result is logical. Option implied ambiguity is indeed more relevant in times of uncertainty.

5 Conclusions

This paper examines the lead–lag relationship between option and stock markets during the subprime crisis, testing the role of option implied ambiguity in volatility and risk prediction. We find a positive moderating role of option implied ambiguity in the lead–lag IV–RV relationship. Ambiguity predicts and contributes to positive changes in stock market volatility in highly uncertain times. We document shifts in ambiguity aversion among put option investors in the period leading to the 2008 crash, and show that ambiguity-adjusted estimates contain more information and are less biased than their risk-neutral Black–Scholes and stochastic volatility counterparts. Specifically, we compare the information efficiency of the standard risk-neutral modeling framework with that of our multiple-priors and Choquet-based option pricing models. We extract investor ambiguity and uncertainty attitudes from observed index option prices to test how well the IV–RV relationship holds under ambiguity aversion and ambiguity-seeking. Our findings suggest that standard (risk-neutral) options forecasts can be improved upon via estimating implied volatilities under c-ignorance and by accounting for option implied ambiguity dynamics in market supervision and risk prediction exercises.

Our results also confirm that option investors do not necessarily follow a unique decision rule in situations involving outcomes with ambiguous prospects, reinforcing Keynes' (1936) assertion on the role of optimism and pessimism in financial markets. Our extended ambiguity models, tested on US data during the turbulent 2006–2008 period, offer a better means of tracking investors' beliefs and uncertainty preferences, reflecting traders' tendency to deviate from risk-neutrality and their propensity to insure themselves against crash occurrences or extreme downside risk eventualities. This underscores the need to consider broader economic models and uncertainty frameworks that go beyond standard principles to encompass abnormal, fuzzy or extreme volatility events impacting on the financial system. Such extended frameworks may help better explain actual investment behavior. Given the forward-looking nature of the derived option-based information, behavioral measures of uncertainty implied from options markets can contribute to better monitoring and tracking of investment sentiment and systemic market fluctuations. These measures can also be used as inputs or complements to current indicators of uncertainty in the macro economy.

Future research utilizing the information content of the IC factor and ambiguity-based IV can examine longer time periods and cover other crisis situations (e.g., the 1987 crash, 1997 Asian financial crisis, 2011–2012 Eurozone sovereign debt crisis) using monthly non-overlapping data or intraday information. Assessing the impact of governmental interventions and periodic fiscal stimuli on the evolution of the IC factor also merits attention. Call option model variants can also be analyzed to examine investor ambiguity aversion or ambiguity-seeking with respect to upside (growth) opportunities, or to study volatility smiles, skews and smirks under ambiguity. Both stochastic and model-free volatility dynamics (e.g., Faria and Correia-da-Silva 2014) are also worth investigating from a similar perspective.

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References

Agliardi, E., & Sereno, L. (2011). The effect of environment taxes and quotas on the optimal timing of emission reductions under Choquet-Brownian uncertainty. *Economic Modelling*, *28*, 2793–2802.

Anderson, E. W., Ghysels, E., & Juergens, J. L. (2009). The impact of risk and uncertainty on expected returns. *Journal of Financial Economics*, *94*, 233–263.

Araujo, A., Chateauneuf, A., & Faro, J. H. (2012). Pricing rules and Arrow–Debreu ambiguous valuation. *Economic Theory*, *49*, 1–35.

Ayadi, M. A., Ben-Ameur, H., Kirillov, T., & Welch, R. (2014). A stochastic dynamic program for valuing options on futures. *Journal of Futures Markets*, *34*, 1185–1201.

Bates, D. S. (2008). The market for crash risk. *Journal of Economic Dynamics and Control*, *32*, 2291–2321.

Beber, A., Breedon, F., & Buraschi, A. (2010). Differences in beliefs and currency risk premiums. *Journal of Financial Economics*, *98*, 415–438.

Ben Ameur, H., & Prigent, J. L. (2013). Optimal portfolio positioning under ambiguity. *Economic Modelling*, *34*, 89–97.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, *81*, 637–654.

Boyarchenko, N. (2012). Ambiguity shifts and the 2007–2008 crisis. *Journal of Monetary Economics*, *59*, 493–507.

Buraschi, A., & Jiltsov, A. (2006). Model uncertainty and option markets with heterogeneous beliefs. *Journal of Finance*, *61*, 2841–2897.

Chateauneuf, A., Kast, R., & Lapied, A. (1996). Choquet pricing for financial markets with frictions. *Mathematical Finance*, *6*, 323–330.

Chateauneuf, A., Kast, R., & Lapied, A. (2001). Conditioning capacities and Choquet integrals: The role of comonotony. *Theory and Decision*, *51*, 367–386.

Chateauneuf, A., Eichberger, J., & Grant, S. (2007). Choice under uncertainty with the best and worst in mind: Neo-additive capacities. *Journal of Economic Theory*, *51*, 538–567.

Cherubini, U. (1997). Fuzzy measures and asset prices: accounting for information ambiguity. *Applied Mathematical Finance*, *4*, 135–149.

Christensen, B. J., & Prabhala, N. R. (1998). The relation between implied and realized volatility. *Journal of Financial Economics*, *50*, 125–150.

Clark, E., & Gosh, D. K. (2004). *Arbitrage, hedging and speculation*. Westport, CT: Praeger Publishers.

Clark, E., & S. Baccar. (2015). Modelling credit spreads with time volatility, skewness, and kurtosis. *Annals of Operations Research*, 1–31. doi:10.1007/s10479-015-1975-5.

Cochrane, J. H. (2001). *Asset pricing*. Princeton, NJ: Princeton University press.

Cogley, T., Colacitto, R., Hansen, L. P., & Sargent, T. J. (2008). Robustness and US monetary policy experimentation. *Journal of Money, Credit and Banking*, *40*, 1599–1623.

Dana, R. (2002). On equilibria when agents have multiple priors. *Annals of Operations Research*, *114*, 105–115.

De Bondt, W., & Thaler, R. (1987). Further evidence on investor overreaction and stock market seasonality. *Journal of Finance*, *42*, 557–581.

De Palma, A., Ben-Akiva, M., Brownstone, D., Holt, C., Magnac, T., McFadden, D., et al. (2008). Risk, uncertainty and discrete choice models. *Marketing Letters*, *19*, 269–285.

De Waegenaere, A., & Wakker, P. (2001). Nonmonotonic Choquet integrals. *Journal of Mathematical Economics*, *36*, 45–60.

De Waegenaere, A., Kast, R., & Lapied, A. (2003). Choquet pricing and equilibrium. *Insurance: Mathematics and Economics*, *32*, 359–370.

Doob, J. L. (1953). *Stochastic processes*. Hoboken: Wiley.

Drechsler, I. (2013). Uncertainty, time-varying fear and asset prices. *Journal of Finance*, *68*, 1843–1889.

Driouchi, T., Trigeorgis, L., & Gao, Y. L. (2015). Choquet-based European option pricing with stochastic (and fixed) strikes. *Operations Research Spectrum*, *37*, 787–802.

Eichberger, J., & Kelsey, D. (1999). E-capacities and the Ellsberg paradox. *Theory and Decision*, *46*, 107–140.

Elkhodiry, A., Paradi, J., & Seco, L. (2011). Using equity options to imply credit information. *Annals of Operations Research*, *185*, 45–73.

Faria, G., & Correia-da-Silva, J. (2012). The price of risk and ambiguity in an intertemporal general equilibrium model of asset prices. *Annals of Finance*, *8*, 507–531.

Faria, G., & Correia-da-Silva, J. (2014). A closed-form solution for options with ambiguity about stochastic volatility. *Review of Derivatives Research*, *17*, 125–159.

Feldman, D. (2007). Incomplete information equilibria: Separation theorems and other myths. *Annals of Operations Research*, *151*, 119–149.

Ford, J., Kelsey, D., & Pang, W. (2014). Information and ambiguity: Herd and contrarian behaviour in financial markets. *Theory and Decision*, *75*, 1–15.

Franke, G., Stapleton, R. C., & Subrahmanyam, M. G. (1999). When are options overpriced? The Black–Scholes model and alternative characterisations of the pricing kernel. *Review of Finance*, *3*, 79–102.

Fung, J. K. W. (2007). The information content of option implied volatility surrounding the 1997 Hong Kong stock market crash. *Journal of Futures Markets*, *27*, 555–574.

Gilboa, I., & Schmeidler, D. (1989). Maxmin expected utility with a nonunique prior. *Journal of Mathematical Economics*, *18*, 141–153.

Gilboa, I., & Schmeidler, D. (1994). Additive representations of non-additive measures and the choquet integral. *Annals of Operations Research*, *52*, 43–65.

Gollier, C. (2008). Discounting with fat-tailed economic growth. *Journal of Risk and Uncertainty*, *37*, 171–186.

Gollier, C., & Schlee, E. (2011). Information and the equity premium. *Journal of the European Economic Association*, *9*, 871–902.

Guidolin, M., & Rinaldi, F. (2013). Ambiguity in asset pricing and portfolio choice: A review of the literature. *Theory and Decision*, *74*, 183–217.

Han, B. (2008). Investor sentiment and option prices. *Review of Financial Studies*, *21*, 387–414.

Harrison, J. M., & Kreps, D. M. (1979). Martingale and arbitrage in multiperiod securities markets. *Journal of Economic Theory*, *20*, 381–408.

Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, *6*, 327–343.

Hey, J. D., Lotito, G., & Maffioletti, A. (2010). The descriptive and predictive adequacy of theories of decision-making under uncertainty/ambiguity. *Journal of Risk and Uncertainty*, *41*, 81–111.

Jackwerth, J. C. (2000). Recovering risk aversion from option prices and realized returns. *Review of Financial Studies*, *13*, 433–451.

Jahan-Parvar, M. R., Liu, H. (2014). Ambiguity aversion and asset prices in production economies. *Review of Financial Studies*. doi:10.1093/rfs/hhu037.

Jaimungal, S., & Sigloch, G. (2012). Incorporating risk and ambiguity aversion into a hybrid model of default. *Mathematical Finance*, *22*, 57–81.

Javanmardi, L., Lawryshyn, Y. (2015). A new rank dependent utility approach to model risk averse preferences in portfolio optimization. *Annals of Operations Research*, 1–16. doi:10.1007/s10479-014-1761-9.

Kast, R., & Lapied, A. (2010). Valuing future cash flows with non separable discount factors and non additive subjective measures: Conditional Choquet capacities on time and on uncertainty. *Theory and Decision*, *69*, 27–53.

Kast, R., Lapied, A., & Roubaud, D. (2014). Modelling under ambiguity with dynamically consistent Choquet random walks and Choquet-Brownian motions. *Economic Modelling*, *38*, 495–503.

Kelsey, D., Kozhan, R., & Pang, W. (2011). Asymmetric momentum effects under uncertainty. *Review of Finance*, *15*, 603–631.

Keynes, J. M. (1936). *The general theory of employment, interest and money*. Cambridge: Cambridge University Press.

Kim, J., Leung, T. (2014). Impact of risk aversion and belief heterogeneity on trading of defaultable claims. *Annals of Operations Research*, 1–30. doi:10.1007/s10479-013-1524-z.

Knight, F. H. (1921). *Risk, uncertainty and profit*. Boston, MA: Houghton Mifflin.

Li, T. (2007). Heterogeneous beliefs, asset prices, and volatility in a pure exchange economy. *Journal of Economic Dynamics and Control*, *31*, 1697–1727.

Liu, J., Pan, J., & Wang, T. (2005). An equilibrium model of rare event premia and its implications for option smirks. *Review of Financial Studies*, *18*, 131–164.

Mandelbrot, B., & Hudson, R. L. (2008). *The (mis)behavior of markets: A fractal view of financial turbulence*. New York: Basic Books.

Marzban, S., Mahootchi, M., & Khamseh, A. A. (2015). Developing a multi-period robust optimization model considering American style options. *Annals of Operations Research*, *233*, 305–320.

Mellios, M. (2007). Interest rate options valuation under incomplete information. *Annals of Operations Research*, *151*, 99–117.

Merton, R. C. (1973). Theory of rational option pricing. *Bell Journal of Economics and Management Science*, *4*, 141–183.

Montesano, A. (2008). Effects of uncertainty aversion on the call option market. *Theory and Decision*, *65*, 97–123.

Muzzioli, S. (2010). Option-based forecasts of volatility: An empirical study in the DAX index options market. *European Journal of Finance*, *16*, 561–586.

Muzzioli, S., & Torricelli, C. (2004). A multiperiod binomial model for pricing options in a vague world. *Journal of Economic Dynamics and Control*, *28*, 861–887.

Newey, W. K., & West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, *55*, 703–708.

Nguyen, H. T., Pham, U. H., & Tran, H. D. (2012). On some claims related to Choquet integral risk measures. *Annals of Operations Research*, *195*, 5–31.

Nishimura, K. G., & Ozaki, H. (2007). Irreversible investment and Knightian uncertainty. *Journal of Economic Theory*, *136*, 668–694.

Poteshman, A. (2001). Underreaction, overreaction, and increasing misreaction to information in the option market. *Journal of Finance*, *56*, 851–876.

Riedel, F. (2009). Optimal stopping with multiple priors. *Econometrica*, *77*, 857–908.

Rieger, M. O., & Wang, M. (2012). Can ambiguity aversion solve the equity premium puzzle? Survey evidence from international data. *Finance Research Letters*, *9*, 63–72.

Roubaud, D., Lapied, A., Kast, R. (2010). *Real options under Choquet-Brownian ambiguity*. Greqam working paper.

Schmeidler, D. (1989). Subjective probability and expected utility without additivity. *Econometrica*, *57*, 571–587.

Shefrin, H. (2005). *A behavioral approach to asset pricing*. New York City: Elsevier Academic Press.

Stiglitz, J. E. (2011). Rethinking macroeconomics: What failed and how to repair it. *Journal of the European Economic Association*, *9*, 591–645.

Taylor, S. J., Yadav, P., & Zhang, Y. (2010). The information content of implied volatilities and model-free volatility expectations: Evidence from options written on individual stocks. *Journal of Banking and Finance*, *34*, 871–881.

Trojani, F., & Vanini, P. (2004). Robustness and ambiguity aversion in general equilibrium. *Review of Finance*, *8*, 279–324.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, *5*, 297–323.

Vorbrink, J. (2011). *American options with multiple priors in continuous time*. Bielefeld University working paper.

Wakker, P. P. (2001). Testing and characterizing properties of nonadditive measures through violations of the sure-thing principle. *Econometrica*, *69*, 1039–1059.

Whaley, R. E. (2009). Understanding the VIX. *Journal of Portfolio Management*, *35*, 98–105.

Zagst, R., & Kraus, J. (2011). Stochastic dominance of portfolio insurance strategies. *Annals of Operations Research*, *185*, 75–103.